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IMAGING EXTRA-SOLAR PLANETS WITH AN ULTRA-LARGE SPACE TELESCOPE

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Introduction

NASA's Origins Program is directed toward two main goals: Imaging of galactic evolution in the early universe, and searching for planets orbiting nearby stars.1 The Next-Generation Space Telescope (NGST), operating at low temperature with an 8-m aperture, is well designed to meet the first goal. The goal of imaging planets orbiting nearby stars is more problematic. One line of investigation has been the ULTIMA concept (Ultra-Large Telescope, Integrated Missions in Astronomy).

In this report, I will lay out the resolution requirements for telescopes to achieve the imaging of extrasolar planets, and describe a modeling tool created to investigate the requirements for imaging a planet when it is very near a much brighter star.

Resolution requirements for extrasolar planet imaging

Figure 1 shows the resolution requirements for three tasks related to imaging extrasolar planets. It would appear the HST already has the resolution needed to begin to image the faint blue galaxies, which indeed it has. The HST would also appear to have the capability to separate the images of nearby extrasolar planets from the stars they orbit, which is not the case. The stars are so much brighter than the planets that the diffraction haloes in the images overwhelm the images of the planets. Of the four telescope concepts, none approaches the requirement for imaging details on faces of extrasolar planets. Only the 100-m telescope is capable of imaging a Sun-like star at 10 pc, although a star is bright enough that a filled aperture is not needed for this task.

Imaging a planet orbiting a nearby star

A spreadsheet tool to calculate the signal in a large space telescope from a planet orbiting a nearby star, and compare it with photon noise in the diffraction halo of the star, has been constructed and made available on the Internet (http://www.wou.edu/research/physics/taylorc/taylorc.html).

The distribution of energy with wavelength for a blackbody at temperature T_B is

$$f(\lambda)=\frac{15}{\lambda}\left(\frac{hc}{\pi\lambda k\,T_B}\right)^4\left[e^{\frac{hc}{\lambda kT_B}}-1\right]^{-1}=\frac{2\pi hc^2}{\sigma T_B{}^4\lambda^5}\left[e^{\frac{hc}{\lambda kT_B}}-1\right]^{-1} \qquad (1.)$$

where

$$\sigma=\frac{2\pi^5k^4}{15h^3c^2} \qquad (2.)$$

is the Stefan-Boltzmann constant. The distribution is normalized such that

$$\int_0^\infty d\lambda \ f(\lambda) = 1. \tag{3.}$$

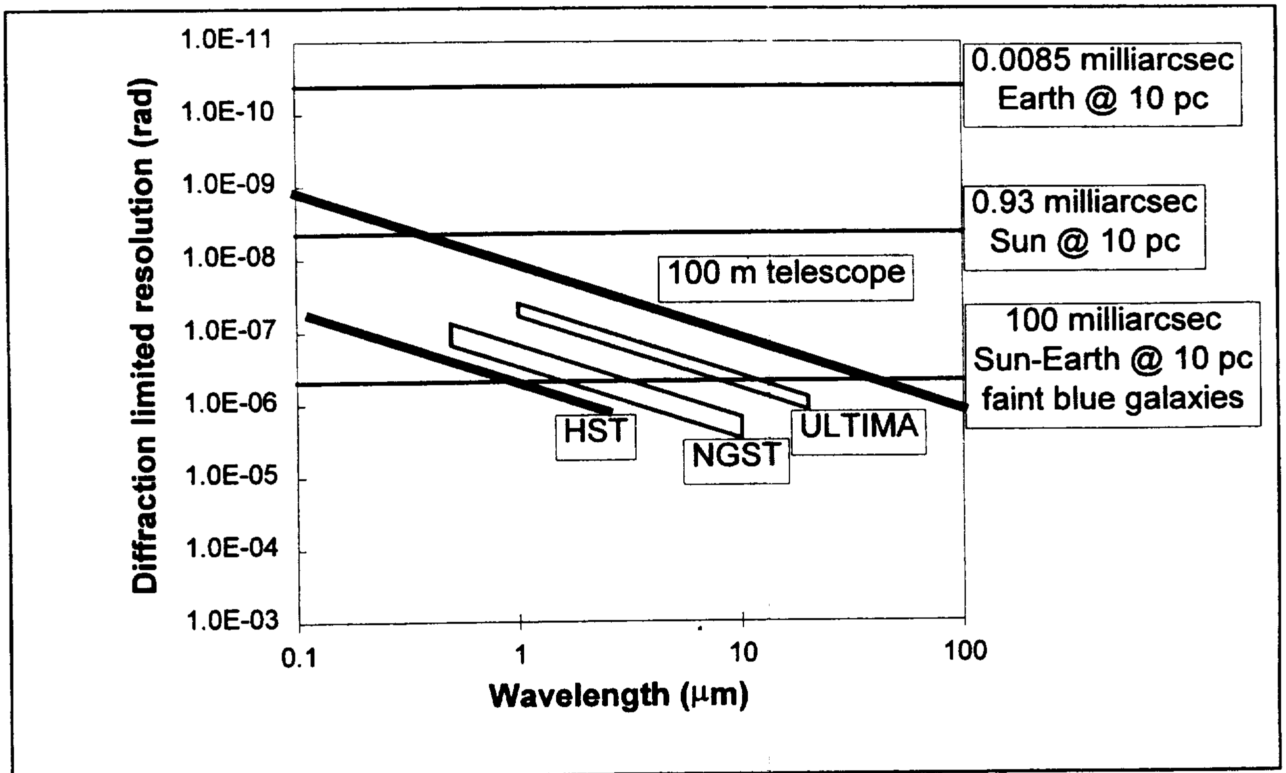

Figure 1. Diffraction limited resolution for telescope concepts and requirements for extrasolar planet imaging. The horizontal lines, starting from the lowest, are the requirements for separating the image of an Earth-like planet from a Sun-like star at 10 pc (which is about the same as the requirement for imaging faint blue galaxies in the early universe), imaging details on the face of the star itself, and imaging details on the face of the planet. The diffraction limited resolution decreases with increasing wavelength, but long wavelengths (near 10 μm) are needed to achieve the goals of the Origins Project.

The distribution of energy in the image of a monochromatic point source in an unobstructed circular aperture of diameter D is given in the Fraunhofer limit by

$$g(u) = \frac{1}{\pi u^2} J_1^{\,2}\left(\frac{\pi D u}{\lambda f}\right) \tag{4.}$$

where u is a radial coordinate centered on the image in the focal plane and f is the focal length.

The energy of a photon of wavelength λ is given by

$$E = \frac{hc}{\lambda} \tag{5.}$$

From equations 1, 4, and 5 it follows that the number of photons from a blackbody point source reaching a pixel is

$$N_B = \phi \tau \int dA \int_0^\infty d\lambda \eta(\lambda) \frac{1}{\pi u^2} J_1^2\left(\frac{\pi D u}{\lambda f}\right) \times \frac{2\pi hc^2}{\sigma T_B^4 \lambda^5} \frac{1}{e^{\frac{hc}{\lambda k T_B}} - 1} \times \frac{\lambda}{hc} \tag{6.}$$

where ϕ is the energy flux on the aperture, τ is the integration time, and η is the throughput of the instrument as a function of wavelength. If we assume a blackbody source of radius R_B at a distance R, the flux is

$$\phi = \frac{\pi}{4} \frac{R_B^2 D^2}{R^2} \sigma T_B^4 \tag{7.}$$

Equation 7 can be rewritten to express the emitted signals of a star and a planet at essentially the same distance.

$$N_S = \frac{\pi}{4} \frac{R_S^2 D^2}{R^2} \tau \int dA \int_0^\infty d\lambda \eta(\lambda) \frac{1}{\pi u^2} J_1^2\left(\frac{\pi D u}{\lambda f}\right) \frac{2\pi c}{\lambda^4} \frac{1}{e^{\frac{hc}{\lambda k T_S}} - 1} \tag{8.}$$

$$N_{PE} = \frac{\pi}{4} \frac{R_P^2 D^2}{R^2} \tau \int dA \int_0^\infty d\lambda \eta(\lambda) \frac{1}{\pi u^2} J_1^2\left(\frac{\pi D u}{\lambda f}\right) \frac{2\pi c}{\lambda^4} \frac{1}{e^{\frac{hc}{\lambda k T_P}} - 1} \tag{9.}$$

The temperature of a planet with no internal energy source can be estimated as

$$T_p = T_s \left[(1-a) \frac{R_s^2}{4r^2} \right]^{\frac{1}{4}} \tag{10.}$$

where a is the albedo and r is its distance from the star.

An equation similar to Equation 9 holds for the light reflected by a planet,

$$N_{PR} = \frac{aR_p^2}{8r^2} \frac{\pi}{4} \frac{R_S^2 D^2}{R^2} \tau \int dA \int_0^\infty d\lambda \eta(\lambda) \frac{1}{\pi u^2} J_1^2\left(\frac{\pi D u}{\lambda f}\right) \frac{2\pi c}{\lambda^4} \frac{1}{e^{\frac{hc}{\lambda k T_S}} - 1} \tag{11.}$$

where r_p is the radius of the planet. Note that the temperature in the wavelength integral is that of the star.

We are interested in the signal from the star at points well removed from the central diffraction peak. The Airy pattern varies greatly across the diffraction rings, but when averaged over a pixel or a wavelength band the variations will be suppressed. The Bessel function at large values of the argument can be approximated by

$$J_1(z) \approx \sqrt{\frac{2}{\pi z}} \cos\left(z - \frac{3\pi}{4}\right). \tag{12.}$$

Averaging over the oscillations of the cosine function, and setting the distribution to a constant near the peak, we have a decent approximation of the entire distribution in the following equation.

$$g(u) \approx \begin{cases} \dfrac{\pi^3}{27}\left(\dfrac{D}{\lambda f}\right)^2 & u < \dfrac{3}{\pi^2}\left(\dfrac{\lambda f}{D}\right) \\ \dfrac{\left(\dfrac{D}{\lambda f}\right)^2}{\left(\dfrac{\pi D u}{\lambda f}\right)^3} & u > \dfrac{3}{\pi^2}\left(\dfrac{\lambda f}{D}\right) \end{cases} . \tag{13.}$$

This approximation is used only for the star signal away from the central peak,

$$N_{S\,halo} \approx \frac{\pi}{4} \frac{R_S{}^2 D^2}{R^2} \tau \frac{2cfw^2}{\pi^2 D u^3} \int_0^\infty d\lambda\, \eta(\lambda) \frac{1}{\lambda^3} \frac{1}{e^{\frac{hc}{\lambda k T_S}} - 1}. \tag{14.}$$

The original variable u has been integrated away; the variable u in equation 14 is the distance of the pixel from the center of the bright star image.

For the planet signal, the central peak is calculated. I approximate the square pixel as a circle of the same area, and take the average of two cases: the center of the image in the center of the pixel, and the center of the image at the corner of four pixels. This yields

$$\begin{aligned} &\int dA \frac{1}{\pi u^2} J_1{}^2\left(\frac{\pi D u}{\lambda f}\right) \approx \\ &\approx \frac{5}{8} - \frac{1}{2} J_0{}^2\left(\frac{\sqrt{\pi} D w}{\lambda f}\right) - \frac{1}{2} J_1{}^2\left(\frac{\sqrt{\pi} D w}{\lambda f}\right) - \frac{1}{8} J_0{}^2\left(\frac{2\sqrt{\pi} D w}{\lambda f}\right) - \frac{1}{8} J_1{}^2\left(\frac{2\sqrt{\pi} D w}{\lambda f}\right) \end{aligned} \tag{15.}$$

The pixel width w is taken to be half of the Airy disk diameter,

$$w = \frac{1.22 f \lambda_o}{D} \tag{16.}$$

for the wavelength λ_0 for which the focal plane is optimized for Nyquist sampling. Practically, this equation will fix the focal length of the instrument based on the chosen aperture and available pixel width.

Note also that the angular separation of the planet and star, on average, is

$$\alpha = \frac{r}{\sqrt{2}R} \tag{17.}$$

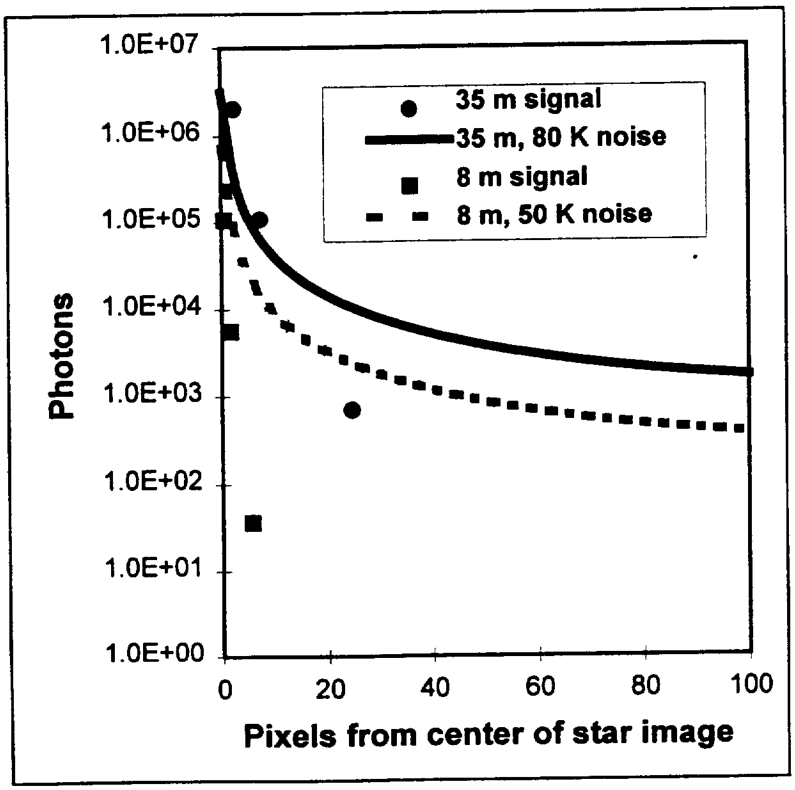

Figure 2. Earth-like extrasolar planet signals and photon noise from nearby Sun-like star, in 35 m and 8 m telescopes. The distance of the star is 5 pc, and the planets are 1,3, and 10 AU from the star.

An example of the results of the calculation is shown in Figure 2. A planet is considered Earth-like if its radius and albedo are equal to those of the Earth. A Sun-like star is one with the same radius and temperature as the Sun. I take the distance to be 5 pc, a distance within which there are about 60 stars. I assume a pixel size of 25 μm optimized for a wavelength of 8 μm, and a band-pass filter to exclude light outside of the range from 10 μm to 20 μm. The likely signal levels from Earth-like planets at 1, 3, and 10 AU are shown for two apertures: 8 m and 35 m. The planet signals are well below the noise at 8 m but above noise levels in the 35 m diameter aperture for the 1 and 3 AU orbits.

Reference

1 A. Dressler, ed., Exploration and the Search for Origins: A Vision for Ultraviolet—Optical-Infrared Space Astronomy, Report of the "HST and Beyond" Committee, Association of Universities for Research in Astronomy, Washington, DC, 1996, http://ngst.gsfc.nasa.gov/project/bin/HST_Beyond.PDF.